Models in Physics

Introduction

Two main activities for philosophers of science

1) Answer Kant's question – How is science possible? How can theories be devised ~~which~~ to explain one group of phenomena and give successful predictions about some other quite different phenomena. – involves Epistemology and Ontology

2.) To describe what exactly goes on in science as accurately as possible, examine the arguments employed, lay bare their logical structure, criticise them for inconsistencies or logical gaps etc. The ~~tool~~ philosophical tool employed here is Logic

First approach tends to use a formal analytical approach that does not need to look at science as it is practised – but prescribes how it ought to be practised.

The second approach looks at science as it actually is, accepts it as a rather successful activity, and analyses the logical interconnections of its arguments. It adopts a very historical approach to philosophy of science

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Methodology
- Appraisal
- heuristics
- conceptual analysis

Theoretical models are inadequate theories
about something we are interested in
Analogue models are adequate theories
about something we are not interested
in.

There are two main types of model used in physics.

theoretical models — set of inadequate theories
analogue models — adequate theories about different objects or systems from those discussed in the theory.

I confine myself to the first because
a) one cannot do everything
b.) Analogue models tend to have disappeared from modern theoretical physics

But Role of analogy in physics has been elevated into a general study of epistemological & ontological questions of how we can provide "satisfactory" explanations, justification of predictive growth of theories etc.

There is divorce here between philosophy of science and science as it is actually practised.

Such studies are clearly legitimate for a normative philosophy of science.

— My plea is that my own more humble approach is not dismissed as an entirely irrelevant pursuit so far as philosophy of science is concerned.

$$\begin{tikzcd}
T_i \arrow[r] \arrow[rd, squiggly] & T_M \arrow[d, squiggly] \\
 & A \\
 & T_A
\end{tikzcd}$$

Models in Physics

Literature : Leatherdale, The Role of Analogy, Model and Metaphor in Science (1974) 200 references

Taxonomy : Rom Harré, 'The Principles of Scientific Thinking' complete with greek names to match – Megamorphs, paramorphs. Teleiomorphs etc.

Theoretical Models: (Black, 1962 Models, Metaphors)

cf Hesse Model, (Models, analogies in science)

Achinstein's Criteria (Concepts of Science 1968)

1.) Set of assumptions about some object or system
2.) Attributes an inner structure, composition or mechanism (manifests itself in other properties exhibited)
3.) Simplified approximations (useful for certain purposes)
4.) Proposed in the framework of a theory
5.) Display an analogy (with some other possibly more familiar object or system)

① is true of theories (on realist account)
② may be true of theories
③ distinguishes theories from models
We believe our theories, tentatively, to be true
A TM is definitely believed to be false.
(cf Popperian account)

④ Stages how models arise:-

(a) Theory T too complicated to solve exactly so TM proposed with a more tractable structure

TM is an impoverished theory

(b) T is incompletely specified by general constraints, symmetry etc.

TM fills in 'missing' detail cp Axiomatic field theory.

TM now enriches the theory

If TM is introduced for its tractability

If TM is seriously entertained → honorific status of theory.

conversely T → TM if it known to be inadequate.

Exs: quark model → quark theory

Early K.T. ~~early [illegible]~~ → Billiard ball model of gas.

Some properties of models are model-independent cp CPT & spin-statistics theorems.

Models & Analogies

$$\begin{aligned} T &: \{P\} = \underset{\text{relevant}}{\{R\}} \cup \underset{\text{non relevant}}{\{N\}} \\ T_M &: \{P'\} = \{R\} \cup \{N'\} \end{aligned}$$

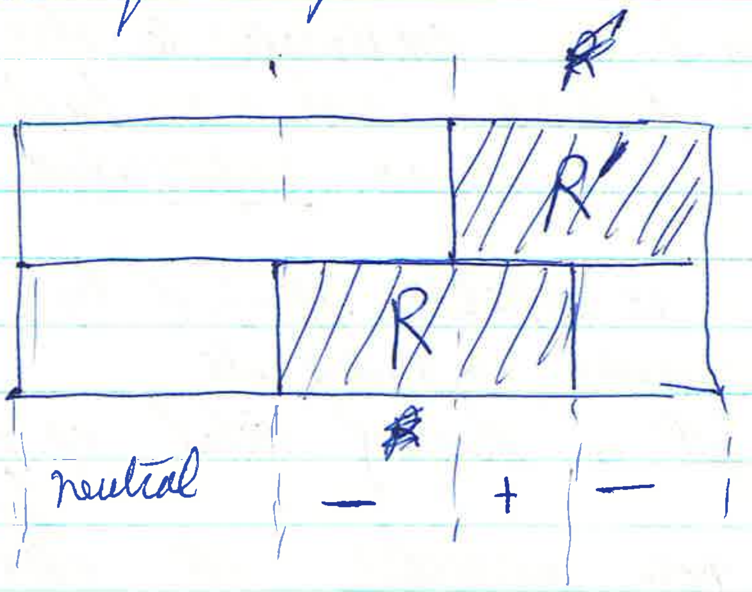
T & TM display material analogy.

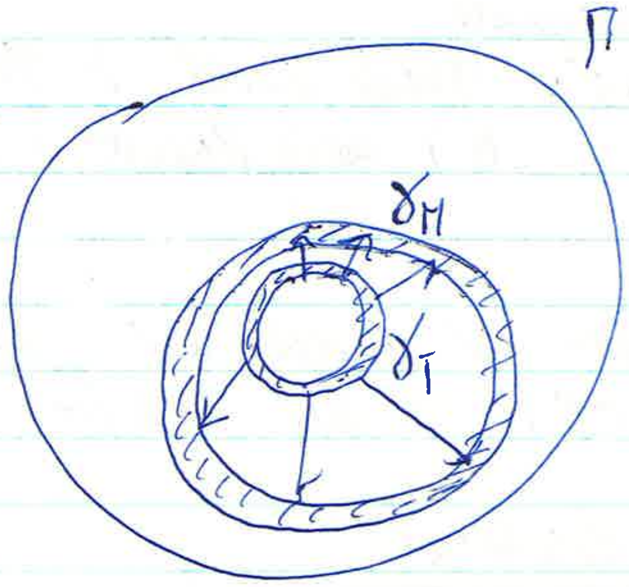
cp.

$$\begin{aligned} T &: \{P\} = \{R\} \cup \{N\} \qquad \{R\} \text{ relevant for } X \\ T_A &: \{P'\} = \{R'\} \cup \{N'\} \qquad \{R'\} \text{ " " } K \end{aligned}$$

T & TA may exhibit only formal analogy (structural isomorphism – related objects possess distinct properties)

* More generally


~~R~~
R'
T_A
R
T
~~R~~
neutral
−
+
−


Γ
γ_M
γ_T

N.B. what is a good approx. depends on what theory is about – cf Miller paradoxes.

$\Gamma = \{f\}$, $f : X \to Y$

(set of all many-one maps from subsets of X into Y (i.e. all functions from X to Y and restrictions of such functions – so $\{f\} \neq Y^X$

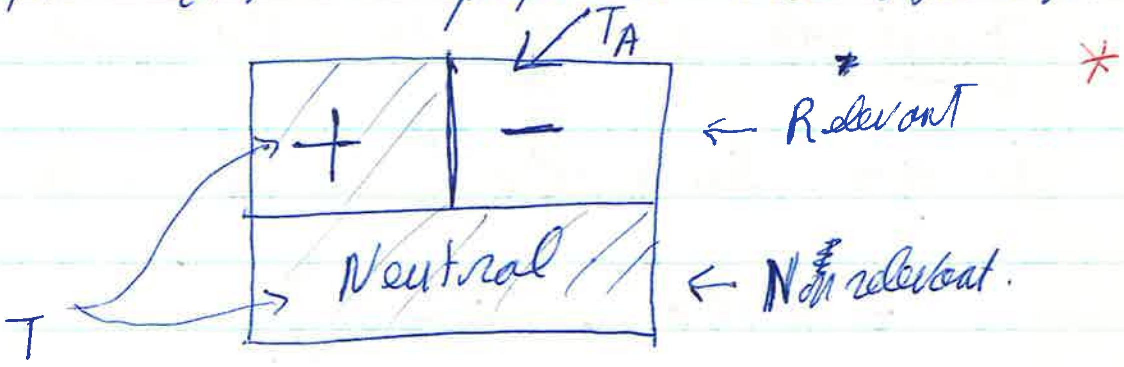
γ_T is subset of Γ permitted by theory T

γ_M " " " " " " " model M

Use of analogy in developing a theory
(weakly predictive in Hesse's sense).

{R'} is hard core in Lakatos sense.
{N'} provide the protective belt.
- can be changed around in accordance with the positive heuristic.

Suppose {R} is a proper subset of {R'} for simplicity



Eg T_A (model$_2$) provides a reservoir of neutral analogies which can be imported into T.

Eg billiard balls can be sticky or non-sticky ((R') is now ascription of inner molecular structure to billiard balls)

Models and approximations

Theories expressed as every relation on some appropriate function space.
Formally we have system of equations where particular solutions give physically possible coordinations of variables.

Two ways of looking at approximations
(a) approximate solutions of the exact equations

*

For general soln $y_n = c\left(1+\lambda x + \cdots \frac{\lambda^{n-1}x^{n-1}}{(n-1)!}\right)$

y_n is a solution

of $$\frac{dy}{dx} - \lambda y \times \left[\frac{1+\lambda x + \cdots \frac{\lambda^{n-2}x^{n-2}}{(n-2)!}}{1+\lambda x + \cdots + \frac{\lambda^{n-1}x^{n-1}}{(n-1)!}}\right] = 0$$

General soln of $\frac{dy}{dx} - \lambda y + \lambda^n x^{n-1}/(n-1)! = 0$

is $y = S_n + Ce^{\lambda x}$ $\quad S_n = 1+\lambda x + \cdots \frac{\lambda^{n-1}x^{n-1}}{(n-1)!}$

$$= (C+1)e^{\lambda x} - (e^{\lambda x} - S_n)$$

$$= \underset{\substack{\uparrow \\ \text{general soln} \\ \text{of } \frac{dy}{dx} - \lambda y = 0}}{c'e^{\lambda x}} + \underset{\uparrow}{(S_n - e^{\lambda x})}$$

correction term (but not to be obtained by iterating on $y(0) = c'$ which yields $c'(S_n - e^{\lambda x})$ as correction term)

(*) So claim is every approximation can be associated with a model.

We are not saying Every apparent model leads to an approximation or that every equation which yields an approximation is a model.

More specifically equations which 'look' similar may have very different solutions and equations which look different may have same solns in common e.g. in variational approach we use full equation (Hamiltonian) to select best of a set of trial solutions (wave functions)

(2) Look for exact solutions to approximate equations. i.e. simplify eq. before we start to solve it

Ex $dy/dx - \lambda y = 0$, small λ, $y=1, x=0$.

we iterate $\frac{dy_n}{dx} = \lambda y_{n-1}$

$$y_n = 1 + \lambda x + \frac{\lambda^2 x^2}{2!} + \cdots \frac{\lambda^{n-1} x^{n-1}}{(n-1)!}.$$

y_n has same value and first $(n-1)$ derivatives at origin as exact solution.
i.e. we have done best we can with $(n-1)^{th}$ degree polynomial; according to this criterion

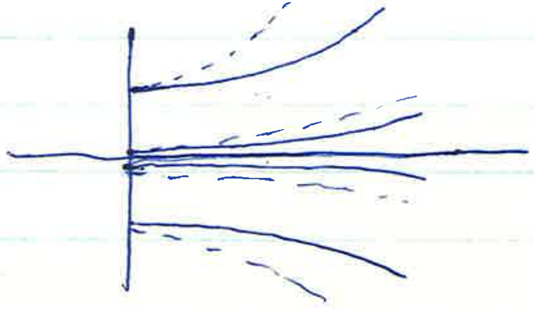
But y_n is also an exact soln of

$$dy/dx - \lambda y + \lambda^n x^{n-1}/(n-1)! = 0.$$

N.B. (1) 'approximate' equations may have quite different solutions due to instability effects – cf hydrodynamic paradoxes – setting viscosity = 0 leads to no resistance to motion of circular cylinder or sphere.

(2) quite different equations may have ~~same~~ approximately the same ~~solutions~~ particular solution

Ex $\frac{dy}{dx} - \lambda y$ and $\frac{dy}{dx} - y = 0$ both have solution $y = 0$



$$(1-x)\frac{dy_3}{dx} = \cos\lambda x + y + O(x^3) \quad *$$

y_3 is exact soln of this eqn

In general models have physical interpretation.
Models generally associated with approximations
Apps in general lack physical significance

Uses of Models

(1) ~~Articulation~~ Probing of Theories

Approximations to a theory may be very misleading

Explore models and see how approximation A_M to model M may represent M — behaviour of M gives clue to how T behaves as against A_T

But A_T is itself equivalent to a model, so by exploring models we may be able to piece together a total picture of how T behaves.

Ex $(1-x)\, dy/dx = \cos \lambda x + y$ $y=0$ at $x=0$

Power series in x gives useful approximation (model) near $x=0$

$$y_3 = x + x^2 + (1-\lambda^2/6)x^3.$$ *

But breaks down near $x=1$.
Small-λ model satisfies eq.

$$(1-x)\, dy/dx = 1 + y$$

Variables are now separated and soln is

$$y_M = x/1-x$$

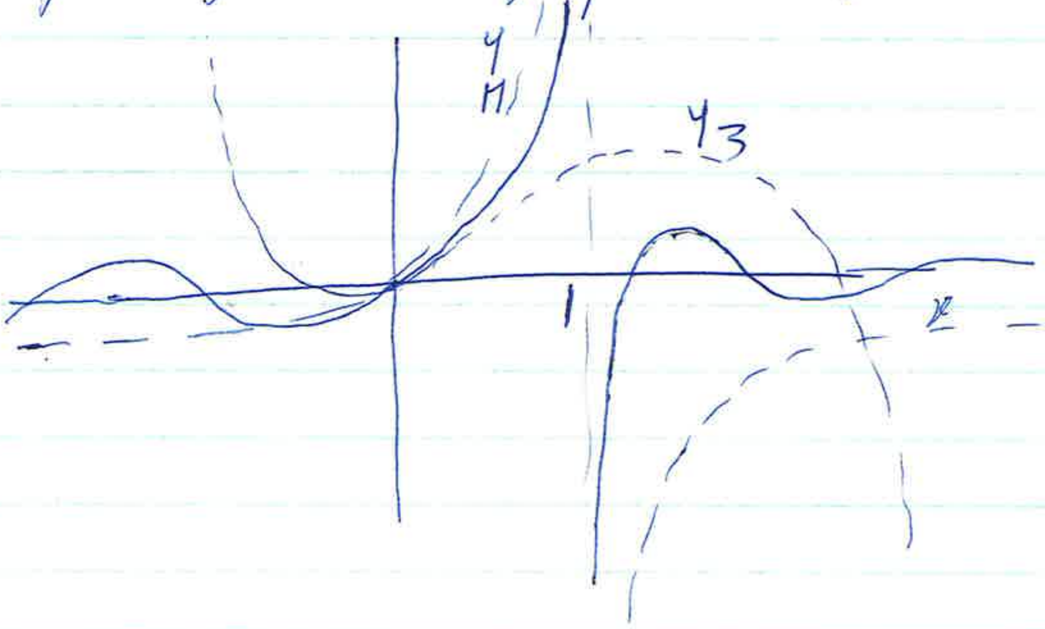
cf expansion $y_M = x + x^2 + x^3 + - -$
no finite no of terms gives pole at $x=1$

Now try $y(x) = \frac{f(x)}{1-x}$.

f(x) satisfies $\frac{df}{dx} = \cos \lambda x$.

$$\therefore \; y = \frac{\sin \lambda x}{\lambda(1-x)}$$

(To a mathematician $(1-x)$ is integrating factor for the original equation)



For $\lambda = n\pi$, $n = 1\,2\,3$ singularity at $x=1$ is cancelled out, so M can still mislead us.

M can direct attention to some aspect of T for particular attention:

But T may be so complicated we do not know if it does behave like M. ⟶

(2) Heuristic Role of Models

Improvement

$$\begin{tikzcd} T' & & \\ T \arrow[r] & M \arrow[r] & P \arrow[ull, bend right] \end{tikzcd}$$

Ex Development of Axiomatic S-matrix theory

Enrichment

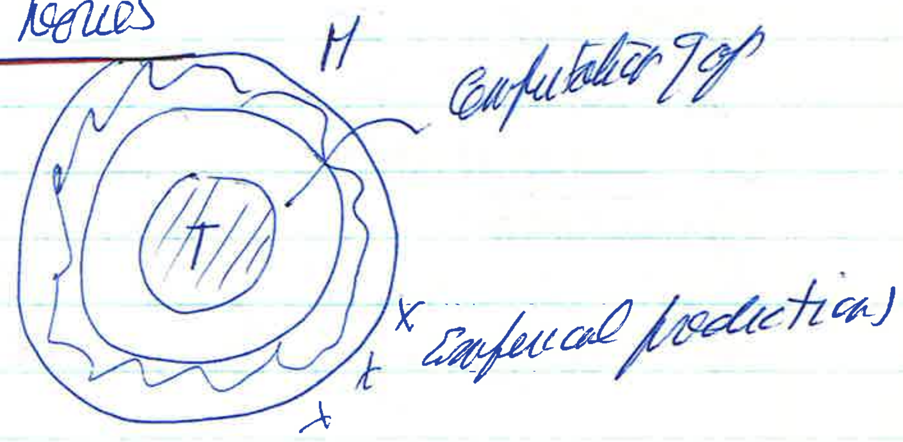
$$\begin{tikzcd} T \arrow[r] & M \arrow[r, dash] & P \arrow[ll, bend right] \end{tikzcd}$$

c now strengthens T

Eg current algebra - derived from quark model of currents.

Post hoc justification for Adler-Weisberger calculation of axial vector coupling constant in β-decay.

(3) Testing of Theories



$$\frac{M \to \neg S \qquad S}{\neg M}$$ by modus tollens

(N.B. A acts on T, not on the world)

But $\neg M \Rightarrow \neg(T \wedge A) \leftrightarrow (\neg T) \vee (\neg A)$

If by mathematics A is a good approximation then S tests T

Or if we believe for independent reasons that T is true S tests A — Empirical mathematics!

Eg N-particle Schrödinger Eq.

✗ $$P_m = \text{Prob}\left(\begin{matrix}\text{prediction}\\ \text{is true}\end{matrix}\middle|\begin{matrix}m\text{-fold}\\ \text{coincidence}\end{matrix}\right) = \frac{x^m}{x^m + \varepsilon(1-x)^m}$$

x = prior prob. of any one model being adequate
ε = prob. coincidence if none of models is adequate

We are interested in enhancement ratio

$$r = \frac{P_m}{P_1} = \frac{1}{x + \dfrac{\varepsilon(1-x)^m}{x^{m-1}}} \simeq \frac{1}{x + \varepsilon/x^{m-1}}$$

$$\varepsilon \simeq 10^{-n(m-1)}\left(= \frac{10^n}{10^{nm}}\right)$$

Assume $x \ll 1$

if $x^m \gg \varepsilon$ $\quad r \gg 1$

But if $x^{m-1} \ll \varepsilon$ $\quad r \ll 1$

So whether we get enhancement or not depends critically on value of x.

The method of coincidence in model predictions is not automatically a sound one for assessing approximations, but depends on our having some reasonable prior confidence in the adequacy of the models.

Another method of testing approximations or models which does not depend on T being true.

$M_1, M_2 \dots$ predict $S_1 \; S_2 \;--$

True value is S.

Define errors $\delta_1, \delta_2 \dots -$

$$S = S_1 + \delta_1 = S_2 + \delta_2 \;--$$

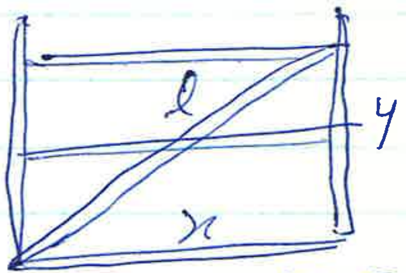
All models adequate $\rightarrow \; \delta_1 = \delta_2 \dots = 0$

$\rightarrow \; S_1 = S_2 = S_3 \quad (= S.)$

Is Reverse implication suggested by Bayesian analysis since it may be quite unlikely that $S_1 = S_2 = S_3$ by mere chance * ?

Floating Models and Disconnected from theory empirical facts.

Ex Four Gate Contraction



linear model: $l = x + y$

instead of $l = \sqrt{x^2 + y^2}$

This match explained by Four Gate Contraction

Ex Elliott model of nucleus.

$$H = H_{osc.} + H_{q-q}$$

$H_{osc.} \downarrow$ SU(3) symmetry

$H_{q-q} \downarrow$ function of generators of SU(3) (non-symmetric)

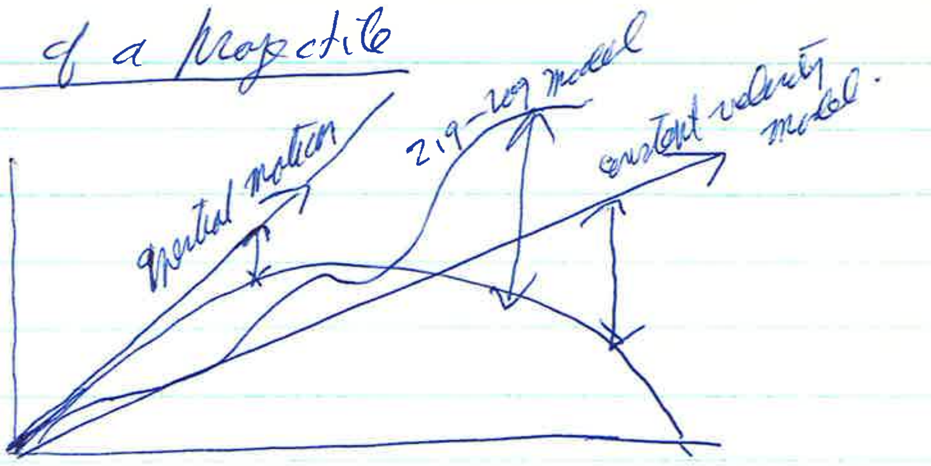
$\Big[H_{q-q} = \sum_{\mu=1}^{5} C_\mu \bar{Q}_\mu \bar{Q}_\mu \quad , \quad \bar{Q}_\mu = \sum_i Q_\mu^{(i)}$

~~Q_μ~~ $Q_\mu = Q_{ij} = T_{ij} - \frac{1}{3} \delta_{ij} \sum_{k=1}^{3} T_{kk}$

$T_{ij} = x_i x_j + p_i p_j \Big]$

this is with special choice of Q_μ more general scalar invariant is $\sum_{\mu,\nu} C_{\mu\nu} Q_\mu Q_\nu \Big]$

Ex Motion of a Projectile

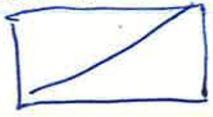


Consider model of inertial motion corrected by gravitational acceleration.
Both components ~~&~~ in analysis of path ~~&~~ corresponding to 'physical' causes.
So inertial model plausibly only at one end

Now consider ~~slanting~~ constant velocity model where effect of gravity is ~~uniform~~ to impart uniform velocity instead of acceleration
This model is equally simple (st. line motion) and mismatch is also simple (diff. between st. line & parabola) so this model is acceptable although components are fictions.
Also consider arbitrary complicated model (zig zag) — can be corrected but correction is also complicated — not acceptable.

Notice however ~~slanting~~ constant velocity & zig zag model are both possible motions — we could produce them with extra forces of rockets etc.
but in Elliott model we cannot physically modify the interaction between nucleons in the way specified in the model.

Homely Ex Sq. Table top can be ~~thought as~~ analysed as 2 Δ'S — but it may not have been made by gluing two Δ's together!



Conclusion

It is tempting to extend the usage of the term model to cover any set of simplified and approximate assumptions about some object or system proposed in the context of some underlying theory. (We drop Achinstein's 2nd criterion – cf usage of mathematical model in Economics etc) Science has been described by Medawar as the Art of the Soluble. Since theories in general are not soluble this can really be translated as science is the Art of Modelling in the extended sense we have proposed. If we always tried to solve every problem with absolute accuracy and neglecting no 'accidental' ~~features~~ aspects, science could never get started. Science depends on the possibility of ignoring accidents, of isolating certain key features in a situation, these are captured by models, although in the very act of idealization and approximation we convince ourselves that the model is indeed false. For Normal Science in the Kuhnean sense we are concerned with a framework of theory (paradigm) which is used for puzzle solving, and in general we do this by making models. Normal Science has sometimes been dismissed, by Popper for example, as a very dull affair. This can be disputed if the intuitive and imaginative elements in model-making are recognized as on a par even with the discovery of new paradigms. Modelling is certainly an Art, although not perhaps the Black Art that some philosophers of science would have us believe.